Worksheet 6 – Rational or Irrational?

1. Reduce these "fat" fractions to lowest terms:
2. Show that the numbers below are rational by expressing each as a ratio of two integers.
3. Show that each of the decimal numbers below is actually a rational number by expressing it as a ratio of two integers.

0.02, 6.23, 2.71828, -168.5, -0.00005

1. Express each fraction in its decimal expansion.
2. Suppose M = 0.499999… Then what does 10M equal? Find two expressions for the quantity 10M – M and set those two expressions equal to each other. (Hint: One expression is simply 9M.) Can you solve your equation to discover something marvelous about M?
3. Express each number as a fraction.
   1. 1.28901
   2. 20.4545
   3. 12.999
   4. 2.2222222…
   5. 43.12121212…
   6. 5.6312121212…
   7. 0.0101010101…
   8. 71.23999999…
4. Find a rational number that is bigger than 12.0345678 and smaller than 12.0345679.
5. Find a rational number that is bigger than 3.14159 and smaller than 3.14159001.
6. Describe an irrational number that is bigger than 5.7 but smaller than 5.72.
7. Is it possible to build an irrational number whose decimal digits are just 1's and 2's? If so, describe such a number and show why it is irrational. If not, explain why.
8. Is it possible to build an irrational number whose decimal digits are just 1's and 2's and only finitely many 2's appear? If so, describe such a number and show why it is irrational. If not, explain why.
9. We know that is irrational. Therefore must also be irrational. Is this conclusion correct? Why or why not?
10. We know that 2/5 and 7/3 are rational. Therefore (2/5)/(7/3) is also rational. Is this conclusion correct? Why or why not?
11. For each of the following numbers, determine if the number is rational or irrational. Why?
12. Prove that is irrational.
13. Show that is irrational.
14. Show that for any prime number *p*, is an irrational number.
15. The which is a rational number. Try using the argument above to prove is irrational. Where does the argument break down?